Exercises for Chapter 6

6.1 Finished part dimensions are measured on a micrometer, and the variability in measurements can be assumed to be a sum of the part to part variance, the variance due to gage, and finally the variance of repeat measurements. Symbolically this is expressed as:

$$\sigma_T^2 = \sigma_P^2 + \sigma_G^2 + \sigma_R^2$$

If the three variance components are respectively: 2.73, 0.034, and 0.132:

a) Calculate the total variance in measured part dimensions

b) Calculate the percentage or proportion of the total variation in measures that is actually due to part to part differences in dimensions.

6.2 Wernimont[1] studied the variability in the measured melting point of hydroquinone. There were 4 different thermometers used in the study, and 2 repeat experiments were performed with each thermometer. The data is shown below:

Thermometer	Measurement	Melting Point
A	1	174.0
A	2	173.0
B	1	173.0
B	2	172.0
C	1	171.5
C	2	171.0
D	1	173.5
D	2	171.0

a) Complete the ANOVA table, and compute the variance components for thermometer and repeat measurement.

b) Make a dot frequency diagram of the data and spot any outliers

[1]Werimont, G. (1947) "Quality Control in the Chemical Industry II. Statistical Quality Control in the Chemical Laboratory," *Industrial Quality Control*, May, p. 8

6.3 Duncan[2] presents data from a rubber-tread experiment. Two mixes were made for each of six different tread formulas. Two samples were taken from the slab cured from the first mix and duplicate tests were run. Only one sample was taken from the second mix, and one test was run.. The data are shown below:

Tread Formula	Mix 1		Mix 2
	Sample 1	Sample 2	Sample 1
1	98	52	86
2	75	96	64
3	34	7	34
4	32	19	7
5	138	113	53
6	102	74	204

a) Write the model for this data. Define each term in your model.

b) With the help of Appendix Table B.7.2-1 complete the ANOVA table and estimate the three components of variance.

c) Make a dot frequency diagram of the data, note any unusual data points and describe how they might affect the estimates.

[2]Duncan, A.J. (1952) *Quality Control and Industrial Statistics*, Richard D. Irwin Inc, Homewood, Ill, p. 614.

6.4 An assembly consists of three stacked parts. Due to manufacturing imperfections the parts are not identical, and their thicknesses follow a normal distribution with a range of natural variation of ±.006 inches (which represents three standard deviations).What tolerance range (i.e., ±3σ limits) could you hope to hold on the thickness of the stacked assembly?

6.5 A finish on a metal consist of 3 coats. A primer coat, a finish coat and a clear coat. The mean and standard deviation of the thicknesses of the three coats are $\mu_P = 0.0025$, $\sigma_P = 0.0013$, $\mu_F = 0.0021$, $\sigma_F = 0.0011$, $\mu_C = 0.0017$, $\sigma_C = 0.0008$. What is the mean and standard deviation of the finish?

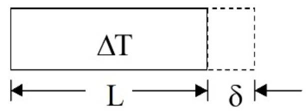
6.6 Injection molded caps are produced for cylindrical metal parts that are purchased from a supplier. The standard deviation of the clearance between the inside diameter of the cap, and the outside diameter of the metal cylinders must be no more than .02 inches if the cap is to fit snugly. If the standard deviation of the outside diameter of the cylinders is .005 inches, what must the standard deviation of the cap inside diameters be in order to insure that the caps to fit snugly?

6.7 A voltage is connected across a resistor. Due to manufacturing imperfections there is variability in both the voltage and the resistance. The resulting current is a random variable whose value is given by Ohm's law as a function of the random voltage and resistance as:

$$I = V/R$$

If the variance of the voltage is $\sigma_V^2 = 23$, and the variance of the resistor is $\sigma_R^2 = 95$, the mean or nominal voltage is 120V, and the mean or nominal resistance is 900Ω. Use the variance propagation formula to find the variance in the current.

6.8 The thermal expansion, δ, is given by the equation , $\delta = \alpha \times L \times \Delta T$ where α is a constant, L is the length, and ΔT is the change in temperature. Assuming $\alpha = 0.031$, and the change in temperature, and length are random variables with means and standard deviations given by: $\mu_L = 54.15$, $\sigma_L = 0.13$, $\mu_{\Delta T} = 150.0$, $\sigma_{\Delta T} = 2.0$. Use the variance propagation formula to calculate the variance of the thermal expansion.



Thermal Expansion

6.9 In a production process, a web of material is extruded and wound on a reel. Periodically a sample of the web is punched out as the material is wound and the thickness is measured and recorded. QA personnel are not sure how much of the variability in recorded thicknesses are actually due to product variability and how much is due to measurement error. The measurement error could be due to differences in gages used to measure the thickness, or due to the inability to get exactly the same value when repeating the measurement with the same gage. Describe how you would collect the data needed to calculate estimates of these three sources of variability.

6.10 Suppose that in the preceding problem the cost of taking a sample from the web is $5, the cost of selecting a gage for measurement is $1, and the cost of measuring the thickness with a gage is $2. If no more than $16 can be spent on getting a measure of thickness, and the variance due to sampling $\sigma^2_S = 0.0326$, the variance due to gages $\sigma^2_G = 0.0017$, and the variance due to repeat measurement with the same gage $\sigma^2_M = 0.0724$, which of the two strategies below gives the most accurate measurement (i.e., smallest variance):

a) take two samples from the web, measure each sample twice with two different gages and average the result as the measurement.

b) take one sample, measure it 5 times with one gage and average the results as the measure.

Would your conclusion change if the variances were $\sigma^2_S = 0.0537$, $\sigma^2_G = 0.0021$, and $\sigma^2_M = 0.0224$?

6.11 Bennett and Franklin[3] presented data showing the variability in the concentration of iron in a standard solution as determined by different analysts. A portion of the data is shown below:

Analyst	Repeat Determination	Concentration of iron
1	1	2.963
1	2	2.996
2	1	2.958
2	2	2.964
3	1	2.956
3	2	2.945
4	1	2.948
4	2	2.960
5	1	2.953
5	2	2.961
6	1	2.941
6	2	2.940
7	1	2.963
7	2	2.928
8	1	2.987
8	2	2.989
9	1	2.946
9	2	2.950
10	1	2.956
10	2	2.947

a) Complete the ANOVA table similar to Figure 6.2, and compute the variance components for analyst and repeat determination.
b) Make a dot frequency diagram of the data and spot any outliers

[3]Bennett, C.A. and Franklin, N.L. (1954) *Statistical Analysis in Chemistry and the Chemical Industry*, John Wiley and Sons, N.Y. p 331.

6.12 Wernimont[1] presented data from an experiment designed to test the homogeneity of the copper content of a series of bronze castings from the same pour. Two samples were taken from each of 11 castings and each sample was analyzed in duplicate.

Casting	Sample	Analysis 1	Analysis 2
1	1	85.54	85.56
1	2	85.51	85.54
2	1	85.54	85.60
2	2	85.25	85.25
3	1	85.72	85.77
3	2	84.94	84.95
4	1	85.48	85.50
4	2	84.98	85.02
5	1	85.54	85.57
5	2	85.84	85.84
6	1	85.72	85.86
6	2	85.81	85.91
7	1	85.72	85.76
7	2	85.81	85.84
8	1	86.13	86.12
8	2	86.12	86.20
9	1	85.47	85.49
9	2	85.75	85.77
10	1	84.98	85.10
10	2	85.90	85.90
11	1	85.12	85.17
11	2	85.18	85.24

a) Write the model for this data. Define each term in your model.

b) With the help of Table B.7.1-2 complete the ANOVA table and estimate the three components of variance.

c) Make a dot frequency diagram of the data, note any unusual data points and describe how they might affect the estimates.

6.13 If you add a second sample for mix 2 in the data for problem 6.3, it results in a full nested design in three components given below. Use the formulas for the sums of squares in Table B.7.1-2, or a statistics program with a GLM ANOVA routine, and the coefficients for the expected mean squares given in Table B.7.1-2 to estimate the three variance components from the full data set. How do they compare to the estimates you made in problem 3?

Tread Formula	*Mix 1*		*Mix 2*	
	Sample 1	*Sample 2*	*Sample 1*	*Sample 2*
1	*98*	*52*	*86*	*122*
2	*75*	*96*	*64*	*50*
3	*34*	*7*	*34*	*26*
4	*32*	*19*	*7*	*-22*
5	*138*	*113*	*53*	*108*
6	*102*	*74*	*204*	*223*